max planck institut
informatik

Universität
des
Saarlandes
FR Informatik

Christoph Weidenbach December 11, 2018

Tutorials for "Automated Reasoning WS18/19"
Exercise sheet 8

Exercise 8.1 (3.74):
Consider the below clause set N, $\Sigma = (\{S\}, \{g, b, a\}, \{P, R\})$, with a KBO ordering where all signature symbols and variables have weight 1 and atoms are compared like terms with precedence $P \succ R \succ g \succ b \succ a$.

$$1 : \neg P(x) \vee P(g(x)) \qquad 2 : \neg P(x) \vee R(x, g(x))$$
$$3 : P(a) \vee P(b) \qquad 4 : \neg R(b, g(b)) \vee P(a)$$

1. Compute $\text{grd}(\Sigma, N)_{\neg R(b,g(b)) \vee P(b)}$, i.e., generate all ground instances of N smaller than $\neg R(b, g(b)) \vee P(b)$ and run the partial model operator.
2. Determine the minimal false ground clause and its productive counterpart and perform the superposition inference step on the respective first-order clauses from N, not on the ground instances.
3. Can $\text{grd}(\Sigma, N')_{\neg R(b,g(b)) \vee P(b)}$ be extended to a model for N by adding further (arbitrarily chosen) ground atoms? If no, provide an argument why there is always at least one false clause for any extension, if yes provide the complete model and give an argument why it is a model.

Exercise 8.2 (3.78):
Refute the following set of clauses by superposition, including all redundancy rules. You can freely choose an ordering and selection function. As usual one sort for everything and x is a variable.

$$1 : \neg Q(x) \vee \neg R(x, g(x)) \qquad 2 : \neg P(x) \vee R(a, x) \qquad 3 : P(g(x))$$
$$4 : \neg P(g(x)) \vee R(x, x) \qquad 5 : S(a) \qquad 6 : \neg S(x) \vee Q(x)$$

Exercise* 8.3 (3.84):
Let N be a finite, satisfiable, saturated first-order Horn clause set. A clause is Horn if it has at

most one positive literal. Furthermore, for each Horn clause $(D \vee L) \in N$ with positive literal L, it contains all variables of the clause, i.e., $\text{vars}(D) \subseteq \text{vars}(L)$. Let C be a non-empty ground clause containing only negative literals. Prove that under these assumptions it is decidable whether $N \cup \{C\}$ is unsatisfiable.

It is not encouraged to prepare joint solutions, because we do not support joint exams.