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**Bromberger/Möhle/Schwarz/Weidenbach** **December 15, 2022**

**Tutorials for "Automated Reasoning WS22/23"**
**Exercise sheet 8**

**Exercise 8.1:**
Refute the following set $N$ of clauses

| | | | |
|---|---|---|---|
| (1) | $P(a,b) \lor P(b,a)$ | (2) | $\neg P(a,b) \lor P(f(b,b),b)$ |
| (3) | $\neg P(b,a) \lor Q(g(a))$ | (4) | $\neg Q(g(a)) \lor P(f(b,b),b)$ |
| (5) | $\neg P(f(b,b),b) \lor \neg P(f(b,b),b)$ | | |

both using KBO and LPO with ground superposition by only applying the inference rules Superposition Left and Factoring:

1. using KBO where all variables and signature symbols have weight 1 and $Q \succ P \succ f \succ g \succ b \succ a$,
2. using LPO with precedence $Q \succ P \succ f \succ g \succ b \succ a$.

**Exercise 8.2:**
Consider again the clause set from Exercise 8.1. This time compute the model $N_{\mathcal{I}}$ both for KBO and LPO:

1. using KBO where all variables and signature symbols have weight 1 and $Q \succ P \succ f \succ g \succ b \succ a$. Compute $N_{\mathcal{I}}$, determine the minimal false clause, perform the respective ground superposition inference, add the result to $N$ yielding $N'$ and compute again $N'_{\mathcal{I}}$,
2. using LPO with precedence $Q \succ P \succ f \succ g \succ b \succ a$. Compute $N_{\mathcal{I}}$, determine the minimal false clause, perform the respective ground superposition inference, add the result to $N$ yielding $N'$ and compute again $N'_{\mathcal{I}}$.

**Exercise 8.3:**
Consider the below clause set $N$, $\Sigma = (\{S\}, \{g, b, a\}, \{P, R\})$, with a KBO ordering where

all signature symbols and variables have weight 1 and atoms are compared like terms with precedence $P \succ R \succ g \succ b \succ a$.

$$\begin{array}{ll} 1 & \neg P(x) \vee P(g(x)) \\ 2 & \neg P(x) \vee R(x, g(x)) \\ 3 & P(a) \vee P(b) \\ 4 & \neg R(b, g(b)) \vee P(a) \end{array}$$

1. Compute $\text{grd}(\Sigma, N)^{\prec \neg R(b,g(b)) \vee P(b)}$, i.e., generate all ground instances of $N$ smaller than $\neg R(b, g(b)) \vee P(b)$ and run the partial model operator.
2. Determine the minimal false ground clause and its productive counterpart and perform the superposition inference step on the respective first-order clauses from $N$, not on the ground instances, resulting in the clause set $N'$.
3. Run the partial model operator on $\text{grd}(\Sigma, N')^{\prec \neg R(b,g(b)) \vee P(b)}$. Can the resulting partial model be extended to a model for $N'$ by adding further (arbitrarily chosen) ground atoms? If no, provide an argument why there is always at least one false clause for any extension, if yes provide the complete model and give an argument why it is a model.
4. Consider the above three steps once more after adding the clause $\neg P(g(b))$ to $N$.

**Exercise* 8.4:**
Let $N$ be a finite, satisfiable, saturated first-order Horn clause set. A clause is Horn if it has at most one positive literal. Furthermore, for each Horn clause $(D \vee L) \in N$ with positive literal $L$, it contains all variables of the clause, i.e., $\text{vars}(D) \subseteq \text{vars}(L)$. Let $C$ be a non-empty ground clause containing only negative literals. Prove that under these assumptions it is decidable whether $N \cup \{C\}$ is unsatisfiable.

It is not encouraged to prepare joint solutions, because we do not support joint exams.