UNIVERSITÄT DES SAARLANDES
FR 6.2 – Informatik
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Lecture "Automated Reasoning"
(Winter Term 2016/2017)

Final Examination

Name: ....

Student Number: ....

Some notes:

- Things to do at the beginning:

  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones.
  Whenever you use a new sheet of paper (including scratch paper), first write your name and student number on it.

- Things to do at the end:

  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your examination text.
  Note: Sheets that are accidentally taken out of the lecture room are invalid.

Sign here: **Good luck!**

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Problem	1	2a	2b	3	4	5	6	7a	7b	7c	8	Σ
Answered?												
Points												

Problem 1 *(Superposition Refutation)* (4 points)
Refute the following set of clauses by superposition, including all redundancy rules. You can freely choose an ordering and selection function. As usual one sort for everything and x, y, z are all variables.

1 $\neg R(x, y) \vee \neg R(y, z) \vee R(x, z)$ 2 $\neg R(x, x)$ 3 $R(x, g(x))$

4 $\neg R(x, y) \vee R(y, x)$

Problem 2 *(Superposition Model Building)* (4 + 2 = 6 points)
Consider the below clause set N over predicate R, function g and constant a with respect to an LPO with precedence $g \succ R \succ a$. As usual one sort for everything and x, y are variables.

$$
\begin{array}{llllll}
1 & \neg R(x,y) \lor R(y,x) & 2 & \neg R(x,x) & 3 & R(x,g(x)) \\
4 & \neg R(g(a),a) & & & &
\end{array}
$$

1. Compute $N_{\mathcal{I}}^{\prec R(g(a),g(a))}$ and determine the minimal false clause.
2. Do the respective superposition inference with the minimal false clause, add it to N giving N' and recompute $(N')_{\mathcal{I}}^{\prec R(g(a),g(a))}$.

Problem 3 *(CDCL)* (5 points)

Check satisfiability of the below propositional clauses using $\Rightarrow_{\text{CDCL}}$.

1	$\neg P4 \vee P3$	2	$\neg P3 \vee P4$	3	$P1 \vee P2 \vee P4$
4	$\neg P3 \vee \neg P4$	5	$\neg P1 \vee \neg P4 \vee P2$	6	$\neg P2 \vee \neg P4 \vee P1$
7	$\neg P1 \vee \neg P2 \vee P4$				

Problem 4 *(CNF)* (6 points)

Transform the following formula into CNF using $\Rightarrow_{\mathrm{ACNF}}$. As usual one sort for everything.

$$\forall x.\exists y.\forall z.\exists u.(R(x,y) \rightarrow (R(g(u),g(z)) \leftrightarrow R(u,z)))$$

Problem 5 *(Tableau)* (4 points)

Prove validity of the following formula using standard Tableau (don't use free-variable Tableau). As usual one sort for everything.

$$[(\exists x.\forall y R(x,y)) \wedge (\forall x, y.(R(x,y) \rightarrow R(y,x)))] \rightarrow \exists x.\forall y.R(y,x)$$

Problem 6 *(Knuth Bendix Completion)* (4 points)

Apply $\Rightarrow_{\mathrm{KBC}}$ to the following set of equations. Choose an appropriate ordering. As usual one sort for everything.

$$E = \{f(g(x), x) \approx h(x),\ f(g(x), h(y)) \approx f(x, y),\ h(a) \approx a\}$$

Problem 7 *(Conjectures)* (2 + 2 + 2 = 6 points)

Which of the following statements are true or false? Provide a proof or a counter example.

1. Let s, t be two terms with unifier σ. Then every term in $\text{codom}(\sigma)$ is a subterm of s or a subterm of t.
2. Let $C \lor A$ and $D \lor \neg A$ be two first-order ground clauses. Let A and $\neg A$ be strictly maximal literals in their respective clauses. Then the clause $C \lor D$, the result of a superposition left inference, is smaller than both parent clauses.
3. Let N be a set of satisfiable ground clauses. Assume N is saturated by superposition up to redundancy where in every clause containing a negative literal, one negative literal is always selected. Then $N_{\mathcal{I}} = \{A \mid A \text{ is a positive unit clause in } N\}$.

Problem 8 *(Standard Unification)* (4 points)

Let s, t be two linear, unifiable terms such that $\text{vars}(s) \cap \text{vars}(t) = \emptyset$. Recall a term is *linear*, if every variable occurs at most once in the term. Let

$$\{s = t\} \Rightarrow^*_{\text{SU}} \{x_1 = l_1, \ldots, x_n = l_n\}$$

be a derivation resulting in the above solved form. Prove that each l_i is either a subterm of s or t.