UNIVERSITÄT DES SAARLANDES
MPI – Informatik
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Lecture “Automated Reasoning”
(Winter Term 2020/2021)

Final Examination

Name: ....

Student Number: ....

Some notes:

- Things to do at the beginning:

  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones.
  Whenever you use a new sheet of paper (including scratch paper), first write your name and student number on it.

- Things to do at the end:

  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your examination text.
  Note: Sheets that are accidentally taken out of the lecture room are invalid.

Sign here: **Good luck!**

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Problem	1	2a	2b	3	4	5	6a	6b	6c	7	Σ
Answered?											
Points											

Problem 1 *(Superposition Refutation)* (6 points)

Refute the following set of clauses via Superposition. You may freely choose an ordering and selection function and apply the well-known simplification rules. As usual variables in different clauses are different, x, y denote variables and f, g functions and a, b are constants.

1 $R(a, b) \vee R(b, a)$
2 $\neg R(f(x, y), b) \vee R(b, f(x, y))$
3 $\neg R(a, x) \vee R(f(x, x), x)$
4 $\neg R(b, x) \vee Q(g(x))$
5 $\neg Q(g(x)) \vee R(f(y, y), b)$
6 $\neg R(y, b) \vee \neg R(b, y)$

Problem 2 *(Superposition Model Building)* (4 + 4 = 8 points)

Consider the below clause set N over predicate R, function f and constant a with respect to an LPO with precedence $f \succ R \succ a$. As usual one sort for everything and x, y are variables.

1	$R(f(x,x),y) \lor R(y,f(x,x))$	2	$\neg R(f(x,x),f(y,y)) \lor \neg R(x,a)$
3	$R(x,f(x,x))$	4	$\neg R(x,x)$

a). Compute $N_{\mathcal{I}}^{\prec \neg R(f(a,a),f(a,a)) \lor \neg R(f(a,a),f(a,a))}$ and determine the minimal false clause.

b). Do the respective superposition inference with the minimal false clause, add it to N giving N' and recompute $(N')_{\mathcal{I}}^{\prec \neg R(f(a,a),f(a,a)) \lor \neg R(f(a,a),f(a,a))}$.

Problem 3 *(Propositional CNF)* (8 points)

Transform the formula

$$[\neg(\neg P \vee (Q \wedge R))] \rightarrow [P \wedge (\neg Q \leftrightarrow \neg R)]$$

into CNF using $\Rightarrow_{\mathrm{ACNF}}$.

Problem 4 *(KBC)* (6 points)

Apply Knuth-Bendix completion ($\Rightarrow_{\mathrm{KBC}}$) to the following set of equations with respect to a KBO where all signature symbols (and variables) have weight 1 and $f \succ g \succ b \succ a$.

$$E = \{f(g(x), y) \approx f(x, y),\ g(f(x, y)) \approx f(x, y),\ g(g(x)) \approx g(x)\}$$

Problem 5 *(CDCL(LRA))* (6 points)

Check whether the following clause set is satisfiable via CDCL(LRA), where you may make use of any procedure introduced in the lecture for the linear rational arithmetic (LRA) part.

$$N = \{y < 5 + x \lor y > 5 + x,\ x \approx z - 3,\ y \leq 3x + 2 - z,\ y - 11 + 3x \geq 2z\}$$

Problem 6 *(Conjectures)* (3 + 3 + 3 = 9 points)

Which of the following statements are true or false? Provide a proof or a counter example.

a). Let N be a first-order clause set without equality. Assume that every clause in N has a strictly maximal literal with respect to some reduction ordering. Then N is satisfiable.

b). Let E be a set of equations where for every equation $l \approx r \in E$, $\text{vars}(r) \subseteq \text{vars}(l)$ and l has strictly more symbols than r. Then KBC terminates on E with a convergent system.

c). Consider the two inequations $x > y + 2$ and $2x < 4y + 3$ and the inequation $2y + 4 < 4y + 3$ obtained via elimination of x. Then all integer solutions of $2y + 4 < 4y + 3$ can be extended to integer solutions for x.

Problem 7 *(Saturated Clause Sets)* (4 points)

Let N be a satisfiable, saturated clause set of first-order clauses without equality. Let S be a clause set such that $N \cup S$ is unsatisfiable. Then in order to refute $N \cup S$ via superposition, no inferences between clauses in N need to be considered.