The Array Fragment with Extensionality

The array first-order array theory consists of (at least) three sorts. An index sort $I$, an array sort $A$ and a value sort $V$. The function $\text{store} : A \times I \times V \to A$ stores values into the array and the function $\text{read} : A \times I \to V$ reads values from the array. The array theory $T_{\text{Array}}$ with extensionality consists then of the three first-order axioms

\[
\forall x_A, y_I, z_V. \text{read}(\text{store}(x, y, z), y) \approx z
\]
\[
\forall x_A, y_I, y'_I, z_V. (y \not\approx y' \rightarrow \text{read}(\text{store}(x, y, z), y') \approx \text{read}(x, y'))
\]
\[
\forall x_A, x'_A. \exists y_I. (\text{read}(x, y) \not\approx \text{read}(x', y) \lor x \approx x')
\]
Similar to the Flattening rule of congruence closure, we also assume here that by the introduction of new constants of the appropriate sorts, there are no nested occurrences of \texttt{read} and \texttt{store}.

\[
\text{FlattS} \quad N[\text{store}(a, s, t)]_{p_1, \ldots, p_k} \Rightarrow_{\text{AF}} N[b/p_1, \ldots, p_k] \cup \{b \approx \text{store}(a, s, t)\}
\]

where $b$ is fresh

\[
\text{FlattR} \quad N[\text{read}(a, s)]_{p_1, \ldots, p_k} \Rightarrow_{\text{AF}} N[b/p_1, \ldots, p_k] \cup \{b \approx \text{read}(a, s)\}
\]

where $b$ is fresh

The Flattening rules are applied at most once to each term \texttt{store}(a, s, t) or \texttt{read}(a, s), respectively.
Then the following inference rules are build in combination with a
decision procedure for ground clauses over EUF, e.g., by
CDCL(EUF):

**StoreRead** \[ N \cup \{ a \approx \text{store}(b, i, l) \} \Rightarrow_{AR} N \cup \{ \text{read}(a, i) \approx l \} \]

**StoreTransD** \[ N \cup \{ a \approx \text{store}(b, i, l), l' \approx \text{read}(a', j) \} \Rightarrow_{AR} N \cup \{ i \approx j \lor \text{read}(a, j) \approx \text{read}(b, j) \} \]
if \[ N \models_{EUF} a \approx a' \]

**StoreTransU** \[ N \cup \{ a \approx \text{store}(b, i, l), l' \approx \text{read}(b', j) \} \Rightarrow_{AR} N \cup \{ i \approx j \lor \text{read}(a, j) \approx \text{read}(b, j) \} \]
if \[ N \models_{EUF} b \approx b' \]

**Ext** \[ N \Rightarrow_{AR} N \cup \{ a \approx b \lor \text{read}(a, i) \not\approx \text{read}(b, i) \} \]
if \( a, b \) are arrays in \( N \) and \( i \) is fresh