5.2.1 Theorem (Superposition Soundness)

All inference rules of the superposition calculus are *sound*, i.e., for every rule $N \uplus \{C_1, \ldots, C_n\} \Rightarrow N \cup \{C_1, \ldots, C_n\} \cup \{D\}$ it holds that $\{C_1, \ldots, C_n\} \models D$.

5.2.2 Definition (Abstract Redundancy)

A clause C is *redundant* with respect to a clause set N if for all ground instances $C\sigma$ there are clauses $\{C_1, \ldots, C_n\} \subseteq N$ with ground instances $C_1\tau_1, \ldots, C_n\tau_n$ such that $C_i\tau_i \prec C\sigma$ for all i and $C_1\tau_1, \ldots, C_n\tau_n \models C\sigma$.
Given a set N of clauses $\text{red}(N)$ is the set of clauses redundant with respect to N.

The concrete redundancy notions from Section 3.13, namely Subsumption, Tautology Deletion, Condensation, and Subsumption Resolution all apply to the superposition calculus for first-order logic with equality as well. In addition, rewriting is the most important redundancy criterion in case of equality.

Unit Rewriting $(N \uplus \{C \vee L, t \approx s\}) \Rightarrow_{\text{SUPE}}$ $(N \cup \{C \vee L[s\sigma]_p, t \approx s\})$

provided $L|_p = t\sigma$ and $t\sigma \succ s\sigma$

5.2.3 Definition (Saturation)

A clause set N is *saturated up to redundancy* if for every derivation $N \setminus \text{red}(N) \Rightarrow_{\text{SUPE}} N \cup \{C\}$ it holds $C \in (N \cup \text{red}(N))$.