UNIVERSITÄT DES SAARLANDES
FR 6.2 – Informatik
Christoph Weidenbach

Lecture “Automated Reasoning”
(Winter Term 2016/2017)

Midterm Examination

Name: ....

Student Number: ....

Some notes:

- Things to do at the beginning:

  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones.
  Whenever you use a new sheet of paper (including scratch paper), first write your name and student number on it.

- Things to do at the end:

  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your examination text.
  Note: Sheets that are accidentally taken out of the lecture room are invalid.

Sign here: **Good luck!**

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Problem	1	2a	2b	2c	3	4	5	6a	6b	6c	7	Σ
Answered?												
Points												

Problem 1 *(Superposition Refutation)* (4 points)

Refute the following clause set by superposition where you may apply the reduction rules Condensation, and Subsumption Resolution. Use the ordering $P_4 \succ P_3 \succ P_2 \succ P_1$. You may also make use of a selection function.

1	$P_2 \lor P_4$	2	$P_1 \lor P_4$	3	$\neg P_2 \lor P_1$
4	$P_1 \lor \neg P_3$	5	$P_4 \lor \neg P_3$	6	$\neg P_3 \lor P_2$
7	$\neg P_1 \lor P_3$	8	$\neg P_4 \lor P_3$	9	$\neg P_1 \lor P_2 \lor P_3$
10	$\neg P_2 \lor \neg P_3$	11	$\neg P_4 \lor \neg P_2 \lor P_3$		

Problem 2 *(Superposition Model Building)* (4 + 2 + 2 = 8 points)

Consider the below clause set with atom ordering $P_5 \succ P_4 \succ P_3 \succ P_2 \succ P_1$.

1	$P_1 \lor P_2 \lor P_2$	2	$\neg P_1 \lor \neg P_2$	3	$\neg P_2 \lor \neg P_3$
4	$P_1 \lor P_3$	5	$P_4 \lor P_5 \lor P_1$	6	$\neg P_4 \lor P_1$
7	$\neg P_4 \lor P_2$	8	$\neg P_5 \lor P_2$	9	$\neg P_5 \lor \neg P_3$

(a) Compute $N_{\mathcal{I}}$.

(b) Determine the minimal false clause in $N_{\mathcal{I}}$. Perform the respective superposition inference on the clause. Add the derived clause to N resulting in N' and compute $N'_{\mathcal{I}}$.

(c) Determine the minimal false clause in $N'_{\mathcal{I}}$. Perform the respective superposition inference on the clause. Add the derived clause to N' resulting in N'' and compute $N''_{\mathcal{I}}$.

Problem 3 *(CDCL)* (7 points)

Check via CDCL whether the below clause set is satisfiable.

1	$P11 \lor P12$	2	$P21 \lor P22$	3	$P31 \lor P32$
4	$P41 \lor P42$	5	$\neg P11 \lor P42$	6	$\neg P42 \lor P11$
7	$\neg P11 \lor \neg P21$	8	$\neg P11 \lor \neg P31$	9	$\neg P31 \lor \neg P41$
10	$\neg P12 \lor \neg P22$	11	$\neg P32 \lor \neg P42$	12	$\neg P12 \lor \neg P32$

Problem 4 *(CNF)* (6 points)

Transform the formula

$$(P \lor ((Q \leftrightarrow \top) \land \neg R)) \lor (P \leftrightarrow (Q \leftrightarrow \bot))$$

into CNF using $\Rightarrow_{\mathrm{ACNF}}$.

Problem 5 *(Tableau)* (4 points)

Prove that the formula

$$((\neg P \vee \neg R) \to Q) \to (\neg Q \to (P \wedge R))$$

is valid using tableau. You may use a tree representation of the tableau.

Problem 6 *(Conjectures)* (2 + 2 + 2 = 6 points)

Which of the following statements are true or false? Provide a proof or a counter example.

1. If $N_{\mathcal{I}} \models N$ then N is saturated up to redundancy.
2. If all clauses in N have at most one positive literal and the CDCL rule Propagate is not applicable to the state $(\epsilon; N; \emptyset; 0; \top)$ then N is satisfiable.
3. If all clauses in N have at most one positive literal and there is no clause in N having only negative literals then $N_{\mathcal{I}} \models N$.

Problem 7 *(CDCL Learning and Superposition)* (4 points)

Consider a reasonable CDCL run

$$(\epsilon; N; \emptyset; 0; \top) \Rightarrow^*_{\mathrm{CDCL}} (L_1 \ldots L_n; N; \emptyset; k; D)$$

where the last applied rule was Conflict and hence $D \notin \{\top, \bot\}$. Consider the atom ordering $\mathrm{atom}(L_1) \prec \mathrm{atom}(L_2) \prec \ldots \prec \mathrm{atom}(L_n)$. Prove that any of the subsequent CDCL Resolve steps until backtracking is a Superposition Left inference with respect to $\prec$, where clauses are always condensed.