Decidable First-Order (Clause) Classes

(i) Shallow Linear Monadic Horn Clauses [Weidenbach, 1999, CADE]

(ii) Guarded Fragment [Harald Ganzinger and Hans de Nivelle, 1999, LICS]

(iii) Monadic Fragment with Equality [Leo Bachmair and Harald Ganzinger and Uwe Waldmann, 1993, Computational Logic and Proof Theory]

Idea of the Decidability Proofs

Show that there are only finitely many superposition inferences with respect to redundancy.

More concretely: show that for any (derived) clause C both the number of variables and the maximal depth of terms can be bound. Then Subsumption and Condensation guarantee termination.

Subsumption $(N \uplus \{C_1, C_2\}) \;\Rightarrow_{\text{RES}}\; (N \cup \{C_1\})$

provided $C_1\sigma \subset C_2$ for some matcher σ

Tautology Deletion $(N \uplus \{C \vee A \vee \neg A\}) \;\Rightarrow_{\text{RES}}\; (N)$

Condensation $(N \uplus \{C\}) \;\Rightarrow_{\text{RES}}\; (N \cup \{C'\})$

where C' is the result of removing duplicate literals from $C\sigma$ for some matcher σ and C' subsumes C

Shallow Linear Monadic Horn Clauses

Clauses are of the form

$$\neg A_1 \lor \ldots \lor \neg A_n \lor B$$

where

(i) clauses may be purely negative (without B) or purely positive (just B)

(ii) B has the form $S(x)$, or $S(c)$, or $S(f(x_1, \ldots, x_m))$ all x_i different

(iii) A_i has the form $S(t)$ for an arbitrary term t

Guarded Fragment

Guarded Fragment (GF) Definition ()

Recursively defined by the following rules (no eqaulity, no function symbols):

(i) Every atom A is in GF

(ii) If ϕ, ψ are in GF so are $\neg\phi$, $\phi \circ \psi$ for $\circ \in \{\wedge, \vee, \rightarrow, \leftrightarrow\}$

(iii) If ϕ in GF and A an atom sich that every free variable of ϕ occurs among the arguments of A, then $\forall\vec{x}.(A \rightarrow \phi)$ and $\exists\vec{x}.(A \wedge \phi)$ are in GF.

Guarded Clauses Lemma ()

A clause C belongs to the CNF of a formula ϕ from GF iff

(i) Every non-ground functional term from C contains all variables if C

(ii) If C is not ground, then there exists a negative literal $\neg A \in C$ that does not contain a non-ground, functional term but all variables of C.

Monadic Fragment with Equality (Löwenheim 1915)

Originally, formulas of the form

$$\{\forall, \exists\}^*\phi$$

and ϕ is quantifier free, no function symbols, only monadic predicates.
Building a CNF we get *flat* clauses where

(i) all atoms are of the form $S(t)$ or $s \approx t$

(ii) there exists a sequence of distict variables auch that any term t is either a variable x_n or of the form $f(x_1, \ldots, x_n)$ for $n \leq m$.

Using an LPO one can guarantee that superposition inferences between flat clauses again result in flat clauses.

Furthermore we add the rule Split where now a superposition state consists of a set M of clause sets:

Split $M \uplus \{N \uplus \{C_1 \vee A_1 \vee C_2 \vee A_2\}\}$

$\Rightarrow_{\text{SUP}}$ $M \cup \{N \cup \{C_1 \vee A_1\}, N \cup \{C_2 \vee A_2\}\}$

where $\text{vars}(C_1 \vee A_1) \cap \text{vars}(C_2 \vee A_2) = \emptyset$