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**Tutorials for “Automated Reasoning”**  
**Exercise sheet 11**

**Exercise 11.1:** (3+3 P)

Prove the following formulas by standard first-order tableaux.

1.  $(\exists x.\forall y.R(x, y)) \rightarrow (\forall y.\exists x.R(x, y))$
2.  $\exists x.(P(x) \rightarrow \forall x.P(x))$

**Exercise 11.2:** (3+3 P)

Prove the following formulas by free-variable first-order tableaux.

1.  $\forall x.\exists y.\forall z.\exists w.(R(x, y) \vee \neg R(w, z))$
2.  $\forall x.(P(a) \wedge (P(x) \rightarrow P(g(x)))) \rightarrow P(g(g(g(a))))$

**Exercise 11.3:** (4 P)

Consider the following logical puzzle

Three friends Pablo, Edvard, and Henri are talking to each other about the art collection of Leonardo. Pablo says: “Leonardo has at least four paintings of Rembrandt.” Edvard says: “No, he has less than four paintings of Rembrandt.” “According me,” says Henri, “Leonardo has at least one Rembrandt.” Only one of the friends is right.

Formulate the puzzle in first-order logic in the sense that any model of the respective first-order formula reveals the question about the number of Rembrandt paintings owned by Leonardo.

Submit your solution in lecture hall E1.3, Room 002 during the lecture on January 27. Please write your name and the date of your tutorial group (Mon, Thu) on your solution.

Joint solutions are not permitted, please submit individually. However, I encourage you working and solving the exercises in a group.