First-Order Logic

First-Order logic is a generalization of propositional logic. Propositional logic can represent propositions, whereas first-order logic can represent individuals and propositions about individuals.

For example, in propositional logic from “Socrates is a man” and “If Socrates is a man then Socrates is mortal” the conclusion “Socrates is mortal” can be drawn.

In first-order logic this can be represented much more fine-grained. From “Socrates is a man” and “All man are mortal” the conclusion “Socrates is mortal” can be drawn.

3.1.1 Definition (Many-Sorted Signature)

A *many-sorted signature* $\Sigma = (\mathcal{S}, \Omega, \Pi)$ is a triple consisting of
a finite non-empty set $\mathcal{S}$ of *sort symbols*,
a non-empty set Ω of *operator symbols* (also called *function symbols*) over $\mathcal{S}$ and
a set Π of *predicate symbols*.

3.1.1 Definition (Many-Sorted Signature Ctd)

Every operator symbol $f \in \Omega$ has a unique sort declaration $f : S_1 \times \ldots \times S_n \to S$, indicating the sorts of arguments (also called *domain sorts*) and the *range sort* of f, respectively, for some $S_1, \ldots, S_n, S \in \mathcal{S}$ where $n \geq 0$ is called the *arity* of f, also denoted with arity(f). An operator symbol $f \in \Omega$ with arity 0 is called a *constant*.

Every predicate symbol $P \in \Pi$ has a unique sort declaration $P \subseteq S_1 \times \ldots \times S_n$. A predicate symbol $P \in \Pi$ with arity 0 is called a *propositional variable*. For every sort $S \in \mathcal{S}$ there must be at least one constant $a \in \Omega$ with range sort S.

3.1.1 Definition (Many-Sorted Signature Ctd)

In addition to the signature Σ, a variable set $\mathcal{X}$, disjoint from Ω is assumed, so that for every sort $S \in \mathcal{S}$ there exists a countably infinite subset of $\mathcal{X}$ consisting of variables of the sort S. A variable x of sort S is denoted by x_S.