The Overall Picture

Application System + Problem
System Algorithm + Implementation
Algorithm Calculus + Strategy
Calculus Logic + States + Rules
Logic Syntax + Semantics

Conflict Driven Clause Learning (CDCL)

The CDCL calculus tests satisfiability of a finite set N of propositional clauses.

I assume that $\bot \not\in N$ and that the clauses in N do not contain duplicate literal occurrences. Furthermore, duplicate literal occurrences are always silently removed during rule applications of the calculus. (Exhaustive Condensation.)

The CDCL calculus explicitely builds a candidate model for a clause set. If such a sequence of literals $L_1, \ldots, L_n$ satisfies the clause set N, it is done. If not, there is a false clause $C \in N$ with respect to $L_1, \ldots, L_n$.

Now instead of just backtracking through the literals $L_1, \ldots, L_n$, CDCL generates in addition a new clause, called *learned clause* via resolution, that actually guarantees that the subsequence of $L_1, \ldots, L_n$ that caused C to be false will not be generated anymore.

This causes CDCL to be exponentially more powerful in proof length than its predecessor DPLL or Tableau.

CDCL State

A CDCL problem state is a five-tuple $(M; N; U; k; D)$ where

M a sequence of annotated literals, called a *trail*,

N and U are sets of clauses,

$k \in \mathbb{N}$, and

D is a non-empty clause or $\top$ or $\bot$, called the *mode* of the state.

The set N is initialized by the problem clauses where the set U contains all newly learned clauses that are consequences of clauses from N derived by resolution.

Modes of CDCL States

$(\epsilon; N; \emptyset; 0; \top)$	is the start state for some clause set N
$(M; N; U; k; \top)$	is a final state, if $M \models N$ and all literals from N are defined in M
$(M; N; U; k; \bot)$	is a final state, where N has no model
$(M; N; U; k; \top)$	is an intermediate model search state if $M \not\models N$
$(M; N; U; k; D)$	is a backtracking state if $D \not\in \{\top, \bot\}$

The Role of Levels

Literals in $L \in M$ are either annotated with a number, a level, i.e., they have the form L^k meaning that L is the k^{th} guessed decision literal, or they are annotated with a clause that forced the literal to become true.

A literal L is of *level* k with respect to a problem state $(M; N; U; j; C)$ if L or $\text{comp}(L)$ occurs in M and the first decision literal left from L ($\text{comp}(L)$) in M is annotated with k. If there is no such decision literal then $k = 0$.

A clause D is of *level* k with respect to a problem state $(M; N; U; j; C)$ if k is the maximal level of a literal in D.

CDCL Rules

Propagate $(M; N; U; k; \top) \Rightarrow_{\text{CDCL}} (ML^{C\vee L}; N; U; k; \top)$
provided $C \vee L \in (N \cup U)$, $M \models \neg C$, and L is undefined in M

Decide $(M; N; U; k; \top) \Rightarrow_{\text{CDCL}} (ML^{k+1}; N; U; k+1; \top)$
provided L is undefined in M

Conflict $(M; N; U; k; \top) \Rightarrow_{\text{CDCL}} (M; N; U; k; D)$
provided $D \in (N \cup U)$ and $M \models \neg D$

Skip $(ML^{C\vee L}; N; U; k; D) \Rightarrow_{\text{CDCL}} (M; N; U; k; D)$
provided $D \notin \{\top, \bot\}$ and comp(L) does not occur in D

Resolve $(ML^{C\vee L}; N; U; k; D \vee \text{comp}(L)) \Rightarrow_{\text{CDCL}}$
$(M; N; U; k; D \vee C)$
provided D is of level k

Backtrack $(M_1 K^{i+1} M_2; N; U; k; D \vee L) \Rightarrow_{\text{CDCL}}$
$(M_1 L^{D\vee L}; N; U \cup \{D \vee L\}; i; \top)$
provided L is of level k and D is of level i.

Restart $(M; N; U; k; \top) \Rightarrow_{\text{CDCL}} (\epsilon; N; U; 0; \top)$
provided $M \not\models N$

Forget $(M; N; U \uplus \{C\}; k; \top) \Rightarrow_{\text{CDCL}} (M; N; U; k; \top)$
provided $M \not\models N$